Open Research Journal of Science and Environment
Vol. 1, No. 1, November 2013, PP: 15 -26
Available online at http://scitecpub.com/Journals.php

Research article

MODELING URANIUM AND PHOSPHOROUS DEPOSITION INFLUENCED DEGREE OF SATURATION IN ORGANIC AND LATERITIC SOIL IN PORT HARCOURT METROPOLIS, NIGER DELTA OF NIGERIA.

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Abstract

Organic and lateritic soil were found to deposit low void ratio compared to other formations in the study area, these has develop lots of accumulation of uranium and phosphorus between the two sample formations. The migrations of these two parameters are influenced by high degree of saturation generated from climatic influences of high rain intensities in the study location. The stratification of the formation through geological setting were found to influences several conditions expressed in the derived solutions, the behaviour of both parameters are influenced by formation variations expressed at every phase of the developed model, the study is imperative because the deposition of uranium and phosphorous has been thoroughly expressed at every stage of the formation, experts in the field will applied these conceptual frame work to prevent the deposition of uranium and phosphorous from further migration in the study area.

Keywords: modeling uranium. Phosphorous degree of saturation, organic and lateritic soil

1. Introduction

A major current scientific challenge is scaling from the functional properties of organisms to processes at the ecosystem and global levels (Enquist et al. 2003; Torsvik and Ovreas 2002; Zak et al. 2006). Microbial respiration is a process that has particular importance in the ecosystem and global scales, representing about half of total CO2 flux from soils (Hanson et al. 2000, Eluozo and Nwaoburu,2013). Furthermore, effects of human-induced climate change on soil microbial communities and their metabolic activities could create potentially devastating feedbacks to the Earth's biosphere (Meir et al. 2006). Biomass made up of fast-growing species respires faster than an equal amount of biomass made up of slow-growing species. Microbes with low growth yields (biomass produced per unit substrate consumed) convert a larger fraction of substrate into CO2 during growth, and so respire faster than efficiently growing organisms. It has been observed that there is an inevitable thermodynamic trade-off between growth rate and yield among heterotrophic organisms (Pfeiffer et al. 2001). Past authors have proposed that two opposing ecological strategies exist at either end of this spectrum: a fast-growing, low yield competitive strategy and a slow growing high yield cooperative strategy (Kreft and Bonhoeffer 2005; Pfeiffer et al. 2001). For microbes, the cooperative, slow, efficient growth strategy is more successful in spatially structured environments such as biofilms (Kreft 2004; Kreft and Bonhoeffer 2005; MacLean and Gudelj 2006; Pfeiffer et al. 2001). With over a billion individual cells and estimates of 104–105 distinct genomes per gram of soil (Gans et al., 2005; Tringe et al., 2005; Fierer et al., 2007b, David et al 2008), bacteria in soil are the reservoirs for much of Earth's genetic biodiversity. This vast phylogenetic and functional diversity can be attributed in part to the dynamic physical and chemical heterogeneity of soil, which results in spatial and temporal separation of microorganisms (Papke and Ward, 2004 Katherineel al 2011). Given the high diversity of carbon (C) – rich compounds in soils, the ability of each taxon to compete for only a subset of resources could also contribute to the high diversity of bacteria in soils through resource partitioning (Zhou et al. 2002). Indeed, Waldrop and Firestone (2004) have demonstrated distinct substrate preferences by broad microbial groups in grassland soils and C resource partitioning has been demonstrated to be a key contributor to patterns of bacterial co-existence in model communities on plant surfaces (Wilson and Lindow, 1994,Eluozo and Nwaoburu,2013).

2. Theoretical back ground

Uranium and phosphorous are physiochemical constituents that deposited in the soil formation either natural origin or manmade in Port Harcourt metropolis.. The structural developments deposited organic and lateritic soil fine sand; deposited high pressures are from Benin structural settings under the influence of such geological origin, it always known to deposit homogeneous formation. Discovering lots of pressure from homogeneous predominant in Port Harcourt city, it is confirmed from hydrogeological studies to deposit organic and lateritic soil influenced by deltaic nature. High degree of void ratio has definitely established an interaction with dispersion rate under the influence of the micropores of deposited strata. Uranium and phosphorus were found to deposit in lateritic and organic formation. Such physiochemical depositions were found to establish a reaction. These conditions subject the

Open Research Journal of Science and Environment
Vol. 1, No. 1, November 2013, PP: 15 -26
Available online at http://scitecpub.com/Journals.php

deposition of phosphorous to establish a fluctuation of inhibition by uranium. Subject to this interface, the established reaction between the two parameters leads to vacillation in fluid flow through degree of saturation. Dispersion influence was found to deposit through high percentage of void ratio in the strata dispersing phosphorous and uranium in organic and lateritic soil formation. This has generated high spread of the contaminants in some regions where degree of saturation increase to migrated phosphorous and uranium to other formation through higher percentage of void ratio. Subject to these challenges, there should better solutions to prevent further migration or dispersion of these contaminants. In line with these factors, mathematical model approach was found suitable to express different influences and ways of preventing such pollution. These factors are considered where a system that captured these conditions was considered and it produced the governing equations stated below.

3. Governing equation

Nomenclature

C	-	Concentration
P_b	-	Bulk density
θ	-	Porosity
S	-	Physiochemical
D	-	Dispersion
V		Velocity
X	-	Distance
T	-	Time

$$V\frac{\partial c}{\partial t} + \frac{P_b}{\theta} S \frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial X^2} - V\frac{\partial c}{\partial X} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

The expression from [11] is the governing equation that represents the system developed to monitor the deposition of uranium and phosphorous in organic and lateritic soil. The expressed governing equation expressed parameters found to influences the accumulation of uranium and phosphorous in organic and lateritic soil. The developed equation is influenced by formation characteristics such as high degree of saturation in study location through deltaic influences from high rain intensities.

Applying physical splitting techniques on equation (1)

$$\frac{P_b}{\theta} S \frac{\partial c_1}{\partial t} = P_b \frac{\partial c_1}{\partial t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

$$\left.\begin{aligned} t = 0, \quad x = 0 \\ C_{(o)} = 0 \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

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$$\left.\frac{\partial c_1}{\partial t}\right| t = 0$$

$$V\frac{\partial c_2}{\partial t} = D\frac{\partial^2 c_2}{\partial X^2} \qquad \text{.................................} \qquad (4)$$

$$\left.\begin{array}{l} t = 0, \quad x = 0 \\ C_{(o)} = 0 \\ \left.\dfrac{\partial c_2}{\partial t}\right|_{t = 0} = 0 \end{array}\right\} \qquad \text{.................................} \qquad (5)$$

$$V\frac{\partial^2 c_3}{\partial t} = -\frac{V\partial c_3}{\partial X} \qquad \text{.................................} \qquad (6)$$

$$\left.\begin{array}{l} t = 0 \\ C_{(o)} = 0 \end{array}\right| t = 0 \Bigg\} \qquad \text{.................................} \qquad (7)$$

$$V\frac{\partial^2 c_4}{\partial X^2} = -V\frac{\partial c_4}{\partial X} \qquad \text{.................................} \qquad (8)$$

$$\left.\begin{array}{l} x = 0 \\ t = 0 \\ C_{(o)} = 0 \end{array}\right\} \qquad \text{.................................} \qquad (9)$$

$$\left.\frac{\partial c_4}{\partial X}\right| x = 0 \qquad \text{.................................} \qquad (10)$$

Applying direct integration on (2)

$$V\frac{\partial c}{\partial t} = \frac{P_b}{\theta}C + K_1 \qquad \text{.................................} \qquad (11)$$

Again, integrate equation (11) directly, yield

$$VC = \frac{P_b}{\theta}Ct + K_1 t \quad K_2 \qquad \text{.................................} \qquad (12)$$

Subject to equation (3), we have

$$VC_o = \quad K_2 \qquad \text{.................................} \qquad (13)$$

And subjecting equation (11) to (3)

$$at \left.\frac{\partial c_1}{\partial t}\right|_{t=0} = 0 \quad C_{(o)} = C_o$$

Yield

$$0 = \frac{P_b}{\theta} C_o + K_2$$

$$\Rightarrow K_2 = -\frac{P_b}{\theta} C_o \qquad \text{.................................} \qquad (14)$$

So that, put (13) and (14) into (13), we have

$$VC_1 = \frac{P_b}{\theta} C_1 t - \frac{P_b}{\theta} C_o t + VC_o \qquad \text{.................................} \qquad (15)$$

$$VC_1 - \frac{P_b}{\theta} C_1 t = VC_o - \frac{P_b}{\theta} C_o t \qquad \text{.................................} \qquad (16)$$

$$\Rightarrow C_1 \left(V - \frac{P_b}{\theta} t \right) = C_o \left(V - \frac{P_b}{\theta} t \right) \qquad \text{.................................} \qquad (16)$$

$$\Rightarrow C_1 = C_o \qquad \text{.................................} \qquad (17)$$

Hence equation (16) entails that at any given distance, *x*, we have constant concentration of the contaminant in the system. The expression implies that organic and lateritic formation may develop homogeneous in void ratio, such condition may influences the generation of constant flow of fluid in the formations, the derived expression considered this condition on the derived process as stated in derived solution above.

Now we consider equation (4) which is the progressive phase of the system.

$$V \frac{\partial c_2}{\partial t} = D \frac{\partial^2 c_2}{\partial X^2} \qquad \text{....................} \qquad (4)$$

Approach this system using the Bernoulli's method of separation of variables

$$\text{i.e.} \quad C_2 = XT \qquad \text{.................................} \qquad (18)$$

$$\text{i.e.} \ V \frac{\partial c_2}{\partial t} = XT^1 \qquad \text{.................................} \qquad (19)$$

$$\frac{\partial^2 c_2}{\partial X^2} = X^{11} T \qquad \text{.................................} \qquad (20)$$

Put (19) and (20) into (18), so that we have

$$VXT^1 = DX^{11} T \qquad \text{.................................} \qquad (21)$$

i.e. $\frac{VT^{1}}{T} = \frac{DX^{11}}{X} = -\lambda^{2}$ ……………………………….. (22)

Hence $\frac{VT^{1}}{T} + \lambda^{2} = 0$ ……………………………….. (23)

$$X^{11} + \frac{\lambda^{2}}{V} = 0 \quad \text{……………………………….. (24)}$$

And

$$DX^{11} + \lambda^{2}T = 0 \quad \text{……………………………….. (25)}$$

From (24) $T = ACos\frac{\lambda}{V}t + BSin\frac{\lambda}{V}x$ ……………………………….. (26)

And (19) gives:

$$T = C\ell^{\frac{-\lambda^{2}}{V}t} \quad \text{……………………………….. (27)}$$

By substituting (25) and (26) into (18) we get:

$$C_{2} = \left[ACos\frac{\lambda}{\sqrt{V}}t + BSin\frac{\lambda}{\sqrt{V}}x\right]C\ell^{\frac{-\lambda^{2}}{V}t} \quad \text{……………………………….. (28)}$$

The expressions in [28] explain the condition of uranium and phosphorous deposition at high concentration, this condition implies that the two parameters in organic and lateritic soil will be in exponential stage. Base on these factors, the derived solution were able to direct the expression considering these condition, more so the expression detail the circumstances of phosphorous inhibiting the uranium deposition to increase any microbial contaminant in the system.

Subject equation (28) to condition in (5), so that we have

$$C_{o} = AC \quad \text{……………………………….. (29)}$$

Equation (29) becomes:

$$C_{2} = C_{o}\ell^{\frac{-\lambda^{2}}{D}t}\, Cos\frac{\lambda}{\sqrt{V}}x \quad \text{……………………………….. (30)}$$

Again at $\left.\frac{\partial c_{2}}{\partial t}\right| = 0, \quad x = 0$

$$t = 0, B$$

Equation (30), becomes:

$$\frac{\partial c_{2}}{\partial t} = \frac{\lambda}{\sqrt{V}}C_{o}\ell^{\frac{-\lambda^{2}}{D}t}\, Sin\frac{\lambda}{V}x \quad \text{……………………………….. (31)}$$

i.e. $0 = -C_o \frac{\lambda}{\sqrt{V}} \; Sin \frac{\lambda}{\sqrt{V}} 0$ ……………………………….. (31)

$C_o \frac{\lambda}{\sqrt{V}} \neq 0$ Considering NKP

Which is the substrate utilization for microbial growth (population), so that

$$0 = -C_o \frac{\lambda}{\sqrt{V}} \; Sin \frac{\lambda}{\sqrt{V}} B \qquad \text{……………………………….. (32)}$$

$$\Rightarrow \frac{\lambda}{\sqrt{V}} = \frac{n\pi}{2}, n, 1, 2, 3 \qquad \text{……………………………….. (33)}$$

$$\Rightarrow \lambda = \frac{n\pi\sqrt{V}}{2} \qquad \text{…………………………… (34)}$$

So that equation (30) becomes

$$C_2 = C_o \ell^{\frac{-n^2\pi^2 V}{2D}t} \; Cos \frac{n\pi\sqrt{V}}{2\sqrt{V}} x \qquad \text{……………………………….. (35)}$$

$$C_2 = C_o \ell^{\frac{-n^2\pi^2 V}{2D}t} \; Cos \frac{n\pi}{2} x \qquad \text{……………………………….. (36)}$$

We consider equation (6)

$$V \frac{\partial c_3}{\partial t} = -V \frac{\partial c_3}{\partial X} \qquad \text{……………… (6)}$$

We approach the system by using the Bernoulli's method of separation of variables

$$C_3 = X^1 T \qquad \text{……………………………….. (37)}$$

$$\frac{\partial c_3}{\partial t} = XT^1 \qquad \text{……………………………….. (38)}$$

$$\frac{\partial c_3}{\partial X} = X^1 T \qquad \text{……………………………….. (39)}$$

Again, we put (38) and (39) into (37), so that we have

$$VXT^1 = VX^1 T \qquad \text{……………………………….. (40)}$$

i.e. $\frac{VT^1}{T} = \frac{VX^1}{X} = -\lambda^2$ ……………………………….. (41)

Hence $\frac{VT^1}{T} + \lambda^2 = 0$ ……………………………….. (42)

i.e. $X^1 + \frac{\lambda^2}{V} X = 0$ ………………………………. (43)

And $VT^1 + \lambda^2 T = 0$ ………………………………. (44)

From (44) $X = ACos\frac{\lambda}{\sqrt{V}} X + BSin\frac{\lambda}{\sqrt{V}} X$ ………………………………. (45)

And (38) give

$$T = C\ell^{\frac{-\lambda^2}{V}t}$$ ………………………………. (46)

By substituting (45) and (46) into (37), we get

$$C_3 = \left(ACos\frac{\lambda}{\sqrt{V}} x + BSin\frac{\lambda}{\sqrt{V}} x \right) C\ell^{\frac{-\lambda}{V}t}$$ ………………………………. (47)

Subject (47) to conditions in (9), so that we have

$$C_o = AC$$ ………………………………. (48)

$\therefore$ Equation (48) becomes:

$$C_3 = C_o \ell^{\frac{-\lambda^2}{V}t} Cos\frac{\lambda}{\sqrt{V}} x$$ ………………………………. (49)

Again, at $\left.\frac{\partial c_3}{\partial t}\right|_{t = 0, B} = 0, \quad t = 0$

Equation (49), becomes:

$$\frac{\partial c_3}{\partial t} = \frac{\lambda}{\sqrt{V}} C_o \ell^{\frac{-\lambda^2}{D}t} Sin\frac{\lambda}{V} x$$ ………………………………. (50)

i.e. $0 = \frac{-C_o \lambda}{\sqrt{V}} Sin\frac{\lambda}{V} 0$ ………………………………. (51)

$C_o \frac{\lambda}{\sqrt{V}} \neq 0$ Considering NKP

Which is the substrate utilization for microbial growth (population), so that

$$0 = -C_o \frac{\lambda}{\sqrt{V}} Sin\frac{\lambda}{\sqrt{V}} B$$ ………………………………. (51)

$$\Rightarrow \frac{\lambda}{\sqrt{V}} = \frac{n\pi}{2}$$ ………………………………. (52)

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$$\Rightarrow \lambda = \frac{n\pi\sqrt{V}}{2} \qquad \ldots\ldots\ldots\ldots \qquad (53)$$

So that equation (30) becomes

$$C_3 = C_o \ell^{\frac{-n^2\pi^2 V}{4D}t} \; Cos \frac{n\pi\sqrt{V}}{2\sqrt{V}} x \qquad \ldots\ldots\ldots\ldots \qquad (54)$$

$$\Rightarrow C_3 = C_o \ell^{\frac{-n^2\pi^2 V}{4V}t} \; Cos \frac{n\pi}{2} x \qquad \ldots\ldots\ldots\ldots \qquad (55)$$

The increase of microelements at some region of the formation were considered more in the phase of the derived solution, such condition implies that the deposition of microelements phosphorous can assumed to generate a higher concentration in some region and may at the same time decrease in concentration, such circumstances, the derived expression consider this situation through the integration of microelement in the derived solution as expressed above state equation

Now, we consider equation (8), which is the steady flow rate of the system

$$\frac{D\partial^2 c_4}{\partial X^2} = -V \frac{\partial c_4}{\partial X} \qquad \ldots\ldots\ldots \qquad (8)$$

Using Bernoulli's method, we have

$$C_4 = XT \qquad \ldots\ldots\ldots\ldots \qquad (56)$$

$$\frac{\partial c_4}{\partial X^2} = X^{11}T \qquad \ldots\ldots\ldots\ldots \qquad (57)$$

$$\frac{\partial c_4}{\partial X} = X^1 T \qquad \ldots\ldots\ldots\ldots \qquad (58)$$

Put (57) and (58) into (8), so that we have

$$DX^{11}T = -VX^1 T \qquad \ldots\ldots\ldots\ldots \qquad (59)$$

i.e. $$\frac{DX^{11}}{X} = \frac{VX^1}{X} = \varphi \qquad \ldots\ldots\ldots\ldots \qquad (60)$$

$$\frac{DX^{11}}{X} = \varphi \qquad \ldots\ldots\ldots\ldots \qquad (61)$$

$$\frac{-VX^1}{X} = \varphi \qquad \ldots\ldots\ldots\ldots \qquad (62)$$

$$X = A\frac{\varphi}{D} X \qquad \ldots\ldots\ldots\ldots \qquad (63)$$

And $X = B\ell \dfrac{-\varphi}{V} X$ ……………………………….. (64)

Put (63) and (64) into (56), gives

$$C_4 = A\ell^{\frac{\varphi}{V}x} \quad B\ell^{\frac{-\varphi}{V}x} \qquad \text{……………………………….. (65)}$$

$$C_4 = AB\ell^{(x-x)} \frac{\varphi}{V} \qquad \text{……………………………….. (66)}$$

Subject equation (66) and (67) yield

$$C_{(4)} = (o) = C_o \qquad \text{……………………………….. (67)}$$

So that, equation (68) becomes

$$C_4 = C_o \ell^{(x-x)} \frac{\varphi}{V} \qquad \text{……………………………….. (68)}$$

The steady state of the equation fluctuates, the condition are base on the structural setting of the formation, base on these factors, formation influences plays major roles in the when the fluid flow are in steady state, it implies that the deposition of uranium and phosphorus may be influenced by this soil structural setting in the study area. Therefore the expression integrated these conditions as it stated in the equation above.

Now assuming that, at the steady flow, there is no NKP for substrate utilization, our concentration here is zero, so that equation (68) becomes

$$C_4 = 0 \qquad \text{……………………………….. (69)}$$

There are some conditions whereby the deposition of substrate are inhibited by uranium therefore the expression at equation [69] are denoted as zero, these implies that there is no substrate utilizations, so there the tendency of degradation of any contaminant and uranium increase by its concentrations as inhibiting any microbes if found in the formation, the expression in equation [69] are in line with this condition as it is stated above.

Therefore solution of the system is of the form

$$C = C_1 + C_2 + C_3 + C_4 \qquad \text{……………………………….. (70)}$$

We now substitute (17), (36), (55) and (69) into (70), so that we have the model of the form

$$C = C_o + C_o \ell^{\frac{-n^2\pi^2 V}{2D}t} \bullet \frac{n^2\pi^2 V}{4D}x \; Cos \frac{n^2\pi^2}{4} x \qquad \text{……………………………….. (71)}$$

$$\boxed{\Rightarrow \; C = C_o \left[1 + \ell^{\frac{-n^2\pi^2 V}{2D}t} \bullet \frac{n^2\pi^2 V}{4D}x \; Cos \frac{n^2\pi^2}{4} x \right]} \qquad \text{…………………… (72)}$$

The developed model in [72] showcase the expressed derive solution in the study area, the developed model equation are developed base on the modified system that generated the governing equation, the expression considered several phase of uranium and phosphorous deposition influenced by structural setting of the formation. this condition are in line with the expressed formation variables that were found to developed lots of variation in fluid flow influencing rates of concentration from uranium and phosphorous. The developed model will monitor the deposition of both parameters in these directions.

4. Conclusion

The deposition of uranium and phosphorous has thoroughly expressed in the study area, modeling the deposition of these two parameters are established by various stratification influences, the study were able to monitor the deposition of uranium and phosphorous in the study location, such physiochemical reaction in both parameters, several conditions that proof the inhibitions of uranium by phosphorous deposition were expressed on the process of deriving the solution, the influential parameters such as degree of saturation were integrated thus express it rates of influences in the study area. The deposition of uranium and phosphorous were to express the rates the substances migration through flow path from high degree of void ratio, this determine the migration rate of uranium and phosphorous between organic and lateritic soil. The study is imperative because experts in the field will applied these concepts to monitor the rate of both parameters and prevent further migration to other formation in the study area.

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